Reduced-Dynamic Technique for Precise Orbit Determination of Low Earth Satellites

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Observations of the global positioning system (GPS) will enable a reduced-dynamic technique for achieving subdecimeter orbit determination of low Earth satelites. With this technique, the transition between satellite states at different observing times is furnished by both a formal dynamic model and observed satellite positional change inferred kinematically from continuous GPS carrier phase data. The relative weighting of dynamic and kinematic information can be freely varied. Covariance studies show that in situations where observing geometry is poor and the dynamic model is good, the model dominates determination of the state transition; where the dynamic model is poor and the geometry strong, carrier phase governs the transition. When neither kinematic nor dynamic information is clearly superior, the reduced-dynamic combination can substantially improve the orbit solution. Guidelines are given here for selecting a near-optimal weighting for the reduced-dynamic solution, and sensitivity of solution accuracy to this weighting is examined.

Introduction

THE U.S. Defense Department's global positioning system (GPS) will be in full operation with at least 18 satellites in orbit by the end of 1992. The two precise radiometric data types available from GPS, P-code pseudorange and continuous carrier phase, will provide accurate positioning for users anywhere on the Earth's surface and in low orbits.[1] Advanced differential techniques incorporating GPS data from a global network of ground reference sites now promise to provide the subdecimeter orbit determination accuracy being sought for a growing number of scientific remote sensing satellites.[2-4] One such satellite is the U.S.-French Ocean Topography Experiment, Topex/Poseidon, to be launched in June of 1992. Topex, which will fly in a circular orbit at an altitude of 1336 km (Ref. 5), has a formal accuracy requirement of 13 cm for the continuous determination of its geocentric altitude; in fact, Topex ocean science would benefit from an altitude accuracy comparable to the 2.5-cm precision of its radar altimeter.

Differential GPS tracking can take many forms. The simplest is relative point positioning using instantaneous differential pseudorange measurements to four or more GPS satellites. The accuracy of this *geometric* relative positioning is limited primarily by measurement noise and GPS ephemeris error, magnified by PDOP (position dilution of precision), which is related to observing geometry.[1] For a low Earth orbiter, instantaneous differential positioning accuracy is expected to approach 1 m (Ref. 6). In applications where more accurate orbit knowledge is needed, a long-arc *dynamic* solution may be suitable. With this approach, the satellite dynamics are constrained by physical models and noisy instantaneous measurements obtained over a period of time are combined to improve precision and to yield greater information on the user state at a single epoch.[2,3] For greatest accuracy, all GPS orbits and some ground receiver positions are also adjusted; all solutions are obtained in a reference frame established by a small set of fixed (unadjusted) ground sites. In the dynamic solution, the transition from satellite states at different measurement times to the state at the solution epoch is furnished by integration of the equations of motion, which are governed by the forces (dynamics) acting on the satellites over the time of interest. Any mismodeling of these dynamics will result in systematic errors in the state solution—errors that tend to grow as the data arc length increases.

In Ref. 4, a long-arc nondynamic (or kinematic) tracking technique is proposed. With this technique, the instantaneous user satellite positions are again determined by differential GPS pseudorange measurements; however, the transition between positions at different times is furnished by the satellite positional change inferred from observations of continuous differential GPS carrier phase, enabling many point position solutions to be smoothed over long data arcs. Since GPS carrier phase can be measured extremely accurately (to 1 mm or better in 1 s), with favorable observing geometry it can provide a nearly ideal state transition. Because the kinematic solution is fundamentally geometric, it is free of dynamic modeling errors. In exchange, however, it has a high sensitivity to the continuously changing observing geometry. To maintain decimeter orbit accuracy, strong observing geometry must be ensured by providing sufficient receiving channels on the orbiter and a sound global network of ground receivers.[4]

Kinematic tracking discards orbit dynamic models and the associated information entirely. That, indeed, is one of its principal attractions. Not only is sensitivity to model errors eliminated but the complexity of the solution process is greatly reduced. Nevertheless, we can expect, in general, that an optimal synthesis of dynamic and kinematic information will offer advantages. In the reduced-dynamic technique, first proposed in Ref. 7, both the dynamic and kinematic methods

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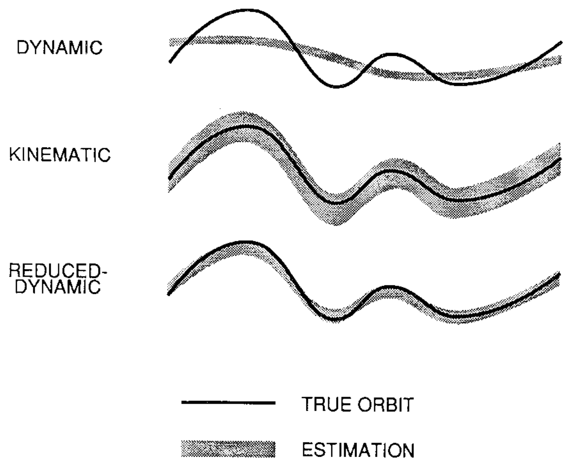

Fig. 1 Qualitative comparison of dynamic, kinematic, and reduced-dynamic tracking performances.

of state transition are used with carefully selected relative weighting. The weight on the dynamic information is controlled through adjusting a set of three process-noise parameters representing a fictitious three-dimensional force on the user satellite. The parameters that vary the relative weighting include the a priori uncertainty σ_0, the steady-state uncertainty σ, and the correlation time τ.

Appending process-noise parameters to the satellite force model is attractive since, although the fictitious force is piecewise constant and therefore discontinuous between batches, the satellite state components, derived by integrating the resulting force model, remain continuous. This fits very naturally with the traditional dynamic formulation. Alternatively, one could add process noise to the satellite state itself (position or velocity). Although this works well in a kinematic solution, it renders the satellite states discontinuous between batches. This complicates incorporation of a (deweighted) dynamic model in a reduced-dynamic formulation and is ill suited to many science applications in which continuous orbits are required.

The dynamic, kinematic, and reduced-dynamic techniques are compared qualitatively in Fig. 1. The dynamic solution adjusts the fewest parameters, preserving maximum data strength and yielding the lowest formal error (error due to data noise), but can suffer a large systematic error from mismodeled dynamics. The kinematic solution eliminates modeling error (the orbit transition is determined entirely from the observations), data strength is depleted, and the formal error can grow large. The reduced-dynamic solution optimally combines the two techniques to achieve the lowest overall error.

We first present a Kalman filter formulation of the reduced-dynamic technique, then describe a covariance analysis carried out to evaluate its performance in comparison with the dynamic and kinematic approaches. Guidelines for selecting a near-optimal reduced-dynamic weighting are given, sensitivity to this weighting is assessed, and practical aspects of the technique are discussed.

Formulation

The reduced-dynamic technique can be described mathematically in terms of Kalman sequential filter formulation. This involves two steps: a time update, which makes use of a state transition model to propagate the satellite state estimate and covariance from one time batch to the next; and a measurement update, which incorporates a new batch of measurements. These two steps alternate until all data batches are incorporated.

Time Update

Let $\hat{x}_j$ be the user satellite state estimate at time t_j using data up to the time t_j, and $\tilde{x}_{j+1}$ the predicted state estimate at time t_{j+1} using data only up to t_j; let $\Phi_x(j+1,j)$ denote the state transition from t_j to t_{j+1}. We now introduce process noise parameters $\boldsymbol{p}$ representing a fictitious three-dimensional force on the user satellite. This gives us the following dynamic (state transition) model for the augmented state $X = [x,p]^0$ and its associated covariance P (Ref. 8):

$$\tilde{X}_{j+1} = \Phi_j \hat{X}_j + Bw_j \tag{1}$$

$$\tilde{P}_{j+1} = \Phi_j \hat{P}_j \Phi_j^0 + BQ_jB^0 \tag{2}$$

where

$$\Phi_j = \begin{bmatrix} \Phi_x(j+1,j) & \Phi_{xp}(j+1,j) \\ 0 & M_j \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} 0 \\ I_p \end{bmatrix} \tag{4}$$

$\Phi_{xp}(j+1,j)$ is the transition matrix relating $\tilde{x}_{j+1}$ to the process noise parameters p_j, M_j is a 3×3 diagonal matrix with its ith element

$$m_i = \exp[-(t_{j+1} - t_j)/\tau_i] \tag{5}$$

w_j is a white noise process of covariance Q_j, which, for convenience, is assumed diagonal with its ith element $q_i = (1 - m_i^2)\sigma_i^2$; I_p is a unit matrix; τ_i is the correlation time constant, which controls the decay rate of the correlation between time batches; and σ_i is the steady-state uncertainty, which is equal to the root-mean-squared (rms) value of the process-noise uncertainty after a long time. Both σ_i and τ_i can be selected to be the same for all i in this application, so we will drop the subscript i. The relative weighting of the dynamics is varied by selecting different values for the steady-state uncertainty σ, the correlation time τ for the process-noise parameters, and the a priori uncertainty σ_0, which is the initial error of the parameters. Increasing τ and decreasing σ_0 and σ increases the weight on the dynamic information. When $\tau \to \infty$, $\sigma \to 0$, and $\sigma_0 \to 0$, the technique reduces to conventional dynamic tracking; when $\tau \to 0$, $\sigma \to \infty$, and $\sigma_0 \to \infty$ it becomes purely kinematic. It follows that an optimal reduced-dynamic solution must be as good as or better than both the purely dynamic and purely kinematic solutions.

Measurement Update

The model for a measurement update in the reduced-dynamic technique is the same as in the dynamic or kinematic techniques, with the exception that X and P are now associated with the augmented state. Thus,

$$\hat{X}_j = \tilde{X}_j + G_j(z_j - A_j\tilde{X}_j) \tag{6}$$

$$\hat{P}_j = \tilde{P}_j - G_jA_j\tilde{P}_j \tag{7}$$

where z_j is the measurement vector at time t_j, A_j is the matrix of the corresponding measurement partials with X_j, and G_j is the Kalman gain given by

$$G_j = \tilde{P}_jA_j^0(A_j\tilde{P}_jA_j^0 + R_j)^{-1} \tag{8}$$

with R_j being the error covariance of z_j.

These models have been formulated in terms of *current* state for clarity. A *pseudoepoch* state, *U-D* factorized formulation[8,9] of these models has been implemented in the GPS analysis software system, OASIS, developed at the Jet Propulsion Laboratory (JPL).[9,10]

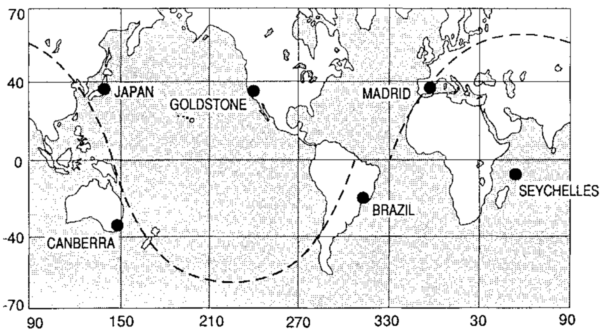

Fig. 2 Ground track of Topex orbit and ground stations used in error analysis.

Covariance Analysis

Assumptions and Approach

Here we present a covariance analysis comparing the accuracy of Topex altitude determination expected with the reduced-dynamic, dynamic, and kinematic techniques. We assume a constellation of 18 GPS satellites placed in six orbit planes. GPS P-code pseudorange and carrier phase data are acquired by a receiver on Topex and by six globally distributed ground sites. Data noise, after a 5-min integration and dual-frequency correction for ionospheric delay, is put at 5 and 0.5 cm for pseudorange and carrier phase, which is consistent with the performance of modern GPS receivers. Carrier phase biases are adjusted with a large a priori uncertainty. A 2-h data arc covering a full Topex orbit is used initially. The ground track of the Topex orbit and the positions of the six ground sites are shown in Fig. 2. Other error sources evaluated are given in Table 1.

The clocks on all GPS satellites and at all ground sites are modeled as white process noise and thus are adjusted independently at each time point. This clock model is comparable to using doubly differenced data and is the most conservative (pessimistic) model we can use because it maximizes the number of adjusted parameters and, hence, the formal error. The gravity error model is derived by scaling the difference between corresponding coefficients, up to degree and order 20, of two different Earth models, GEM10 and GEML2.[11,12] This form of gravity error model is convenient to implement, is easily varied with a single scaling factor, and has proven reliable in numerous studies over the years. Alternatively, one can employ the covariance matrix associated with a single Earth model. Our own experiments have shown that a 50% scaling of GEM10 − GEML2 approximates the error obtained with the covariance matrix from the gravity model, GEMT1,[13] which is one of the best current models.

The 1-cm zenith troposphere error assumes the use of a water-vapor radiometer at each ground site. An earlier analysis showed that the error from mismodeling atmospheric drag is <1 mm for Topex over several-hour arcs of data,[3] therefore, drag is not included here. Such potential error sources as thermal imbalances and outgassing, instrumental delay variations (in the GPS receivers) not common to all signals, and imperfect knowledge of the satellite center of mass (the GPS antenna phase center and platform attitude) are being carefully controlled for Topex and are expected to be below 1 cm. One other potentially serious error source is GPS signal multipath. Because multipath is not readily treated by covariance analysis, separate simulation studies, incorporating all major reflecting surfaces, antenna gain patterns, and receiver tracking characteristics, have been carried out. The result is that, although the instantaneous multipath can at times look alarming (tens of centimeters on pseudorange and up to 1 cm on the carrier), it oscillates with periods of minutes or less and therefore averages down quickly; the net orbit error is typically at the 1-cm level or below after a few hours of averaging.

The inherent GPS data strength allows accurate simultaneous adjustment of the GPS orbits and all but three ground sites.[2-4] Because the GPS satellites are at an altitude where dynamic mismodeling is very small, their solutions remain dynamic in all three approaches. Since we know that GPS orbits will be routinely tracked with high accuracy by a global network, a tight a priori uncertainty is applied to GPS states. By contrast, to examine the strength of each solution technique, a large a priori error is placed on Topex. In actual operation, a much tighter Topex a priori error could be used.

JPL's recently developed Rogue GPS receiver is currently planned for use at all ground reference sites. The Rogue can track up to eight GPS satellites simultaneously, whereas the Topex receiver, built by Motorola, will have a six-satellite capacity. For study purposes, we begin the analysis with the assumption that the Topex and ground receivers can observe all visable GPS satellites (typically six or seven); we then look at cases with lesser Topex receiver capacity. We assess the three tracking techniques by comparing the Topex altitude errors over the entire data span. For this, the pseudoepoch-state covariances of Topex are first smoothed backward and then mapped to all time points when data are taken. Comparison is made between the rms errors calculated over the entire data span.

Results

In a preliminary study not shown here, we examined the limiting cases of the reduced-dynamic technique. With τ set to 0 and both σ_0 and σ set to a large number, the error estimate indeed approached the kinematic solution. With τ set to a

Table 1 Error model and other assumptions used in covariance analysis

User satellite:	TOPEX (1336 km in altitude)
Number of stations:	6 (cf. Fig. 2)
Number of GPS satellites:	18
Cutoff elevation:	10 deg at stations 0 deg at TOPEX
Data type:	P-code pseudorange carrier phase
Data span:	2 h
Data interval:	5 min
Data noise:	5 cm (pseudorange) 0.5 cm (carrier phase)
Carrier phase bias:	10 km (adjusted)
Clock bias:	3 μs (adjusted as white process noise)
TOPEX epoch state:	2 km; 2 m/s (adjusted)
GPS epoch states:	2 m; 0.2 mm/s (adjusted)
Station location:	5 cm each component
Zenith troposphere:	1 cm
Earth's GM:	1 part in 10^8
Gravity:	Scaled GEM10 − GEML2 (see text)
Solar pressure:	10%

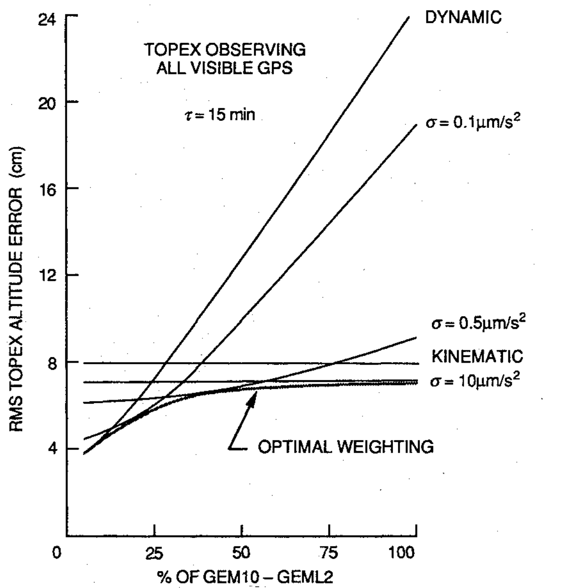

Fig. 3 Performance of reduced-dynamic technique with different weighting on dynamic model.

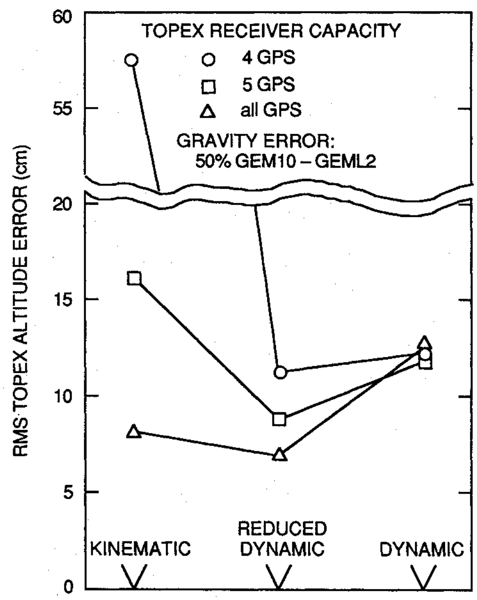

Fig. 4 Performance of dynamic, reduced-dynamic, and kinematic techniques.

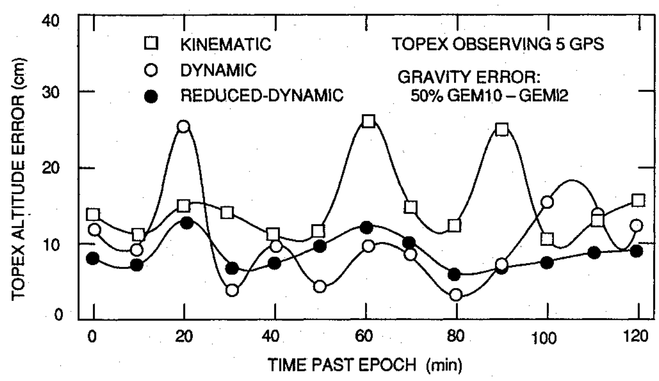

Fig. 5 Time variation of Topex altitude error.

large number and both σ_0 and σ set to 0 it yielded the dynamic estimate. Here we look at a series of intermediate values for τ, σ_0, and σ. In general, when τ is long compared to the batch size, the results vary with the batch-to-batch uncertainty $\sigma_{bb} = (1 - m^2)^{1/2}\sigma$, rather than with the steady-state uncertainty σ and τ individually. (The batch-to-batch uncertainty is the one-sigma change in value from one time batch to the next that is allowed for the process-noise parameters.) In the rest of this analysis, a batch size of 5 min and a constant $\tau = 15$ min are used for all cases with the reduced-dynamic approach; only $\sigma_0 = \sigma$ is varied to yield a nearly optimal solution.

Figure 3 shows the Topex altitude error as a function of the percentage of the GEM10 – GEML2 error, for various values of σ. Also included are the results with dynamic tracking ($\sigma = 0, \tau \rightarrow \infty$) and kinematic tracking ($\sigma \rightarrow \infty, \tau = 0$). It is clear that for any finite dynamic model error (in this case dominated by gravity) a range of σ exists over which Topex altitude error is lower than with either the dynamic or kinematic solutions. In other words, the reduced-dynamic technique is superior provided that the dynamic model is properly weighted. A procedure for estimating the proper weight is outlined later.

Figure 4 compares the reduced-dynamic solution with the dynamic and kinematic solutions for three different viewing capacities for the Topex receiver: four, five, and all GPS satellites (typically six, seldom more than seven) visible above a 90-deg zenith angle. In the cases with restricted receiver capacity, the GPS satellites observed are carefully selected to minimize satellite switches over the observing period (thereby maximizing continuity in carrier phase measurements) while still maintaining good observing geometry (low PDOP). The gravity error is fixed at 50% of the difference between GEM10 and GEML2. A near-optimal weight ($\sigma = 0.5\ \mu m/s^2$) is used for the reduced-dynamic solution in all three cases.

When the Topex receiver can track all visable GPS satellites, the geometry is always strong and kinematic tracking is effective; incorporating the additional dynamic information through the reduced-dynamic technique improves the accuracy by only 1 cm. A lower gravity error, perhaps achieved through gravity tuning, would of course improve reduced-dynamic performance. At the other extreme, when the receiver can track only four GPS satellites, the geometry is often poor and dynamic tracking is far superior to kinematic; the optimal reduced-dynamic combination again offers little advantage. If, however, the gravity error is doubled, as would be the case with a lower orbit, the error with dynamic tracking nearly doubles to 24 cm, while the reduced-dynamic performance degrades only moderately to 16 cm, illustrating that even weak geometry can be of value when dynamics are poorly known. In the intermediate case, with Topex tracking up to five satellites at once, the dynamic and kinematic solutions are better balanced, achieving 12 and 16 cm, respectively. The reduced-dynamic combination improves this to 9 cm. In general, the reduced-dynamic technique is of greatest value when the kinematic and dynamic solutions are comparable.

Dynamic tracking performance degrades over regions where gravity is poorly known (for example, over the oceans). Kinematic performance, on the other hand, can vary dramatically with changing observing geometry. In the reduced-dynamic solution, the two techniques complement one another and the solution is better balanced. This is illustrated in Fig. 5, which compares Topex altitude accuracy over the whole orbit (2 h) using the three techniques. In this case, a Topex receiver tracking five GPS satellites and the 50% GEM10 – GEML2 gravity error are assumed. Both the dynamic and kinematic solutions show peak errors of 25 cm or higher at some points. The reduced-dynamic solution, with a near-optimal weight ($\sigma = 0.5\ \mu m/s^2$), smooths these peaks and remains below 13 cm for the entire period. Reduction of the error peaks results from a balance of state transition information between dynamics and kinematics. When the information from one source is weak, the Kalman filter places greater weight on the other to minimize the overall error. To further illustrate this trade, Fig. 6 breaks down the Topex altitude error into its contributing components at the three epochs when either the dynamic or kinematic error grows large. The balance of transition information in the reduced-dynamic solution has resulted in a more uniform contribution from all error components.

In the examples up to this point, a 2-h tracking arc has been used. In general, as the span is increased, the effects of data noise and troposphere are reduced while the dynamic modeling error grows. In a purely dynamic solution, the effect of increasing model error soon overtakes the decreasing data error and the overall error tends to grow with data span. In the optimal reduced-dynamic solution, however, the estimator continuously shifts weight to the increasingly strong data, away from dynamics, as the span increases. This deweighting of dynamics is a natural consequence of the estimation process; no change in σ is needed since the optimal σ applies to a specific dynamic model error independent of data span. As a result, with optimal weighting the overall performance will tend to improve with increased data span.

Figure 7 compares the Topex altitude error using 2- and 4-h data spans. The longer data span reduces the error over the initial 2-h period to 7 cm from the 8.9 cm obtained with the original 2-h arc. An examination of the error breakdown shows a reduction in gravity error, reflecting the deweighted dynamics, as well as in other errors. Although spans longer than 4 h have not been studied for Topex, we expect that the error will reduce monotonically with data span. Because the weight on the dynamic model decreases with longer data span, a reduced-dynamic solution will tend to a kinematic solution as the span is increased. Note that this is true only on the assumption that a fixed dynamic model is used, independent of the data arc length. If the model is improved through tuning or other efforts, the optimal weight for a given data span will shift back toward the dynamic solution.[14]

Other Applications

Topex nicely illustrates the benefits of reduced-dynamic tracking since its altitude of more than 1300 km, its six-satellite receiver capacity, and its relatively compact dimensions permit both good observing geometry and reasonably well-modeled dynamics. A far greater modeling challenge is presented by several other current or planned NASA space platforms: the large (14-m) platforms of the polar orbiting Earth observing system (Eos), which will carry heavy slewing instruments and fly at 700 km; the actively maneuvering Space Shuttle at altitudes as low as 300 km; and the sprawling (155-m) Space Station Freedom at about 400 km. All will eventually carry experiments seeking tracking accuracies of better than 10 cm. Indeed, a recent international workshop on Space Geodesy set a goal of "no more than 1 cm rms error, single pass, without orbit discontinuities"[15] for tracking future orbiting ocean altimeters, such as the one that will fly on Eos.

Since we cannot expect to approach centimeter or even decimeter accuracy in modeling the dynamics of such ungainly platforms, the optimal orbit solution strategy will be almost purely kinematic. To maximize performance under kinematic tracking, we must maximize geometric observing strength. With that in mind, we have taken the examples of Eos (98-deg inclination, 705-km altitude) and Space Station Freedom (28 deg, 400 km) and carried out covariance studies under a more robust set of assumptions: the GPS constellation is increased to 24 satellites, as is expected to occur by 1995; the ground network is expanded to 10 sites; the flight receiver is extended to track all satellites in view down to the Earth limb (typically a dozen or more); and the three fixed ground sites are assumed known to 2 cm in each component, which is expected to be achieved or surpassed by very long baseline interferometry within the next few years.[16]

Figure 8 plots the rms position errors estimated for a purely kinematic solution, for arc lengths of 2, 4, and 8 h. Under the revised assumptions, the observing geometry is so consistently strong that few-centimeter accuracy is achieved continuously and the rms error approaches 1 cm per component after 8 h. Despite the widely different orbits and dynamics, performance is virtually indistinguishable between Eos and the space station, reflecting the full emphasis on geometric strength (which differs negligibly) over dynamics. We should note that the purely kinematic position solution is referred to the phase

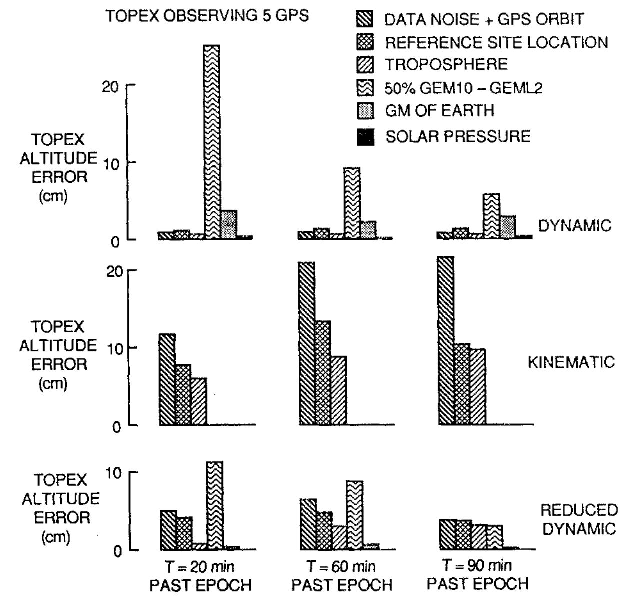

Fig. 6 Breakdown of Topex altitude error at three different epochs.

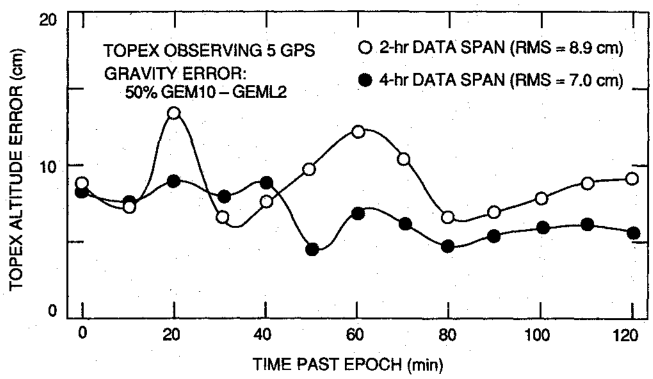

Fig. 7 Comparison of reduced-dynamic tracking performance with different data span.

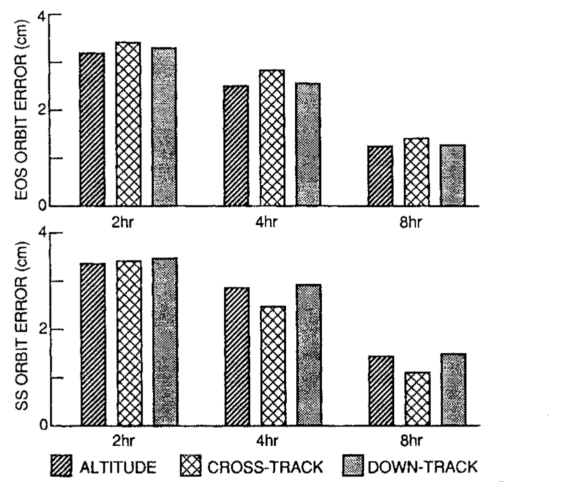

Fig. 8 Estimated kinematic tracking accuracies for Eos and space station with a robust GPS observing system.

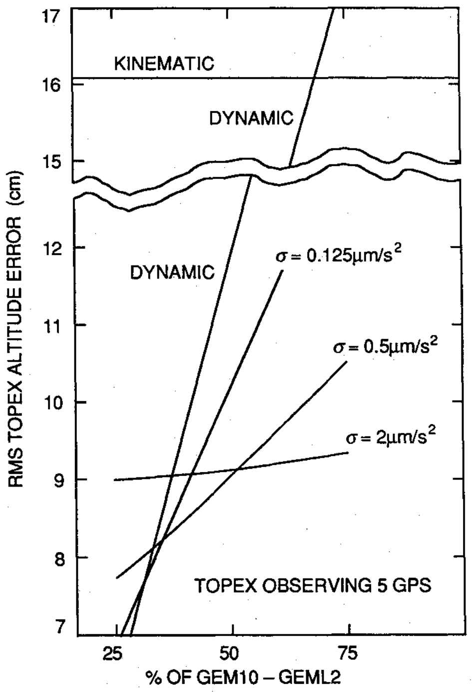

Fig. 9. Insensitivity of Topex accuracy to suboptimal weighting.

center of the orbiter's GPS antenna, which can be calibrated with submillimeter accuracy, rather than to the platform center of mass, which can be difficult to pinpoint on a large and variable structure. Because the dynamics of these orbiters are so poorly known, optimal reduced-dynamic solutions would improve the kinematic results by only 1 or 2 mm—at a great cost in computation.

Weighting the Dynamic Model

For applications like Topex where geometry and dynamics are more balanced, we need a procedure to estimate the weight for the dynamic model, specified by σ with any adopted τ, that minimizes the rms orbit error. This may be difficult to do precisely because the quality of our dynamic models is not always well understood; often, in fact, unsuspected modeling errors are present. Fortunately, the sensitivity to departure from optimal weighting appears to be low; this is illustrated in Fig. 9, in which the Topex altitude error with the reduced-dynamic technique is shown as a function of the level of gravity error for three different weightings. Dynamic and kinematic results are shown for comparison. The weight $\sigma = 0.5\ \mu m/s^2$ is nearly optimal for a gravity error of 50% of GEM10 − GEML2. The two other curves are, for values of σ, a factor of 4 larger ($\sigma = 2\ \mu m/s^2$) and smaller ($\sigma = 0.125\ \mu m/s^2$). This wide range of suboptimal weights increases the Topex altitude error by only 0.3 cm at one end and 1.5 cm at the other. In other words, the performance is fairly insensitive to suboptimal σ.

In practice, an approximate weight can be estimated in advance through a covariance analysis. This is done by a series of filter runs, each using a different weight, simulating the actual measurement and estimation scenario. Realistic data noise and models for all significant (including dynamic) error sources must be considered and their effects on the orbit determination evaluated. The weight resulting in the lowest estimated rms orbit error is the best estimate of the optimal weight for actual data processing.

A misjudgement of the dynamic model error will, of course, result in selection of a suboptimal weight; however, one can take care to minimize the effect of such a misjudgment with the following strategy. The assumed dynamic error model is used first to predict the performance of both the dynamic and kinematic solutions. If either of these appears far superior to the other, say, by a factor of 3 or more, the slight improvement that would result by combining the two approaches with the reduced-dynamic technique may not justify the extra effort and the simpler form can be adopted. If neither technique is far superior, a weight departing from the predicted optimum in a direction favoring the kinematic (i.e., larger σ) will be prudent. Such a bias in the weighting can reduce the more damaging effect of dynamic error in the event it is larger than expected. This point is illustrated in Table 2, which summarizes the effect of weight misjudgment by a factor of 4 on Topex altitude determination for two different levels of gravity error. These results suggest that a weight biased in favor of the kinematic approach is preferable when the level of dynamic error cannot be well determined.

Other Considerations

In the analysis of the reduced-dynamic solution, a fictitious three-dimensional force on Topex was treated as process noise and adjusted together with Topex and GPS states. Its introduction is merely for the purpose of changing the filter model to reduce its reliance on the dynamic model. Since, in the real world, this force does not exist, its presence in the formulation adds an error source in the estimation process, causing the formal error to be overestimated. To remove this effect, an "evaluation" run[17] of the filter is needed. In an evaluation run, the filter model would be specified as before, including the fictitious force, but the contribution of this force in the "truth" model would be ignored. Such an algorithm is fairly complicated when a smoothing process is required because of the dynamic process-noise parameters involved. For a fair estimate of this effect, we have made evaluation runs with the process-noise force replaced by correlated piecewise-constant (in time) forces, thus avoiding the need of smoothing. These runs show that the spurious increase in the formal error due to the fictitious force is only a few millimeters; the corresponding increase in the total error is even smaller, typically 1–2 mm.

Conclusions

A reduced-dynamic technique for determining the orbits of low Earth satellites is made possible with observations of the global positioning system (GPS). In this technique, information on the satellite state transition obtained from both dynamic model and continuous GPS carrier phase observations is optimally combined to improve orbit determination accuracy. Analysis indicates that a significant improvement can be expected when neither of the two types of state transition information is far superior to the other. Performance is not highly sensitive to the relative weighting between dynamic and kinematic information. When the actual level of dynamic model error is uncertain, an additional deweighting of the dynamic model is recommended; this would prevent an inordinately large error resulting from larger-than-expected dynamic model error.

Although a tracking arc of 2 h was used for most of the reduced-dynamic analysis, a 4-h span was examined to illus-

Table 2 Effects of weight misjudgment on TOPEX altitude determination accuracy

Gravity error, %	$\frac{1}{4} \times$ optimal σ, cm	Optimal σ, cm	$4 \times$ optimal σ, cm
50	10.4	8.9	9.2
100	12.3	9.7	10.1

trate the improved performance with increased data. Further improvement can be expected with longer data spans due to reduction of the effects of data noise and random error in tropospheric delay modeling. The effects of increasing dynamic model error will be automatically controlled in the reduced-dynamic solution by further deweighting and will thus remain at a low level. Because the weight on the dynamic model is lowered with growing data strength, a reduced-dynamic solution will gradually approach the kinematic solution as the data span increases, provided that a fixed dynamic model is used.

Reduced-dynamic tracking can be used with any Earth satellite that can adequately observe GPS. The altitude range over which reduced-dynamic tracking provides useful improvement over dynamic and kinematic tracking will depend on the actual level of dynamic model uncertainty. We expect that satellites at altitudes between 400 and 2000 km will receive the greatest benefit. Above this range GPS observability diminishes while dynamic model errors decrease markedly; thus, purely dynamic tracking will be favored. Below this range, uncertainties in gravity and atmospheric drag become so great that a kinematic solution may be favored for simplicity. For some exceptional vehicles, other considerations apply. The actively maneuvering Space Shuttle may receive no benefit from dynamics at any altitude, whereas drag-compensated satellites may exploit dynamics at even the lowest orbit altitudes.

In designing a GPS-based precise tracking system for a low orbiter, we have a simple tradeoff between modeling accuracy and geometric strength to consider. Where the models are strong, the geometry can be relaxed and the flight and ground systems kept relatively simple. Where the models are weak, as will be the case with a number of dynamically complex missions in the future, the geometry must be strengthened. In any case, the global coverage and unique mix of data types offered by GPS ensure that there will be a practical system design and solution strategy that can deliver orbit accuracies well under a decimeter for any low Earth satellite.

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